Can the age discrepancies of neutron stars be circumvented by an accretion-assisted torque?

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ABSTRACT

It is found that 1E 1207.4-5209 could be a low-mass bare strange star if its small radius or low altitude cyclotron formation can be identified. The age problems of five sources could be solved by a fossil-disk-assisted torque. The magnetic dipole radiation dominates the evolution of PSR B1757-24 at present, and the others are in propeller (or tracking) phases.

Subject headings: pulsars: general — pulsars: individual (1E 1207.4-5209) — stars: neutron

1. Introduction

The age of neutron star is an essential parameter, which is relevant to the physics of supernova explosion and thereafter the evolution of stars. However, it is still a big problem now to determine generally an exact value of age (except the crab pulsar). It is a conventional and convenient way to obtain the age for rotation-powered neutron stars by equalizing the energy lose rate of spindown to that of magnetodipole radiation, assuming that the inclination angle between magnetic and rotational axes is $\alpha = 90^{\rm o}$ (e.g., Manchester & Taylor 1977). The conclusion keeps quantitatively for any α, as long as the braking torques due to magnetodipole radiation and the unipolar generator are combined (Xu & Qiao 2001). The resultant age, the so-called characteristic age, is $T_{\rm c} = P/(2\dot{P})$ if the initial period P_0 is much smaller than the present period P. The age, $T_{\rm c}$, is generally considered as the true one for a neutron star with $P > 100$ ms since most newborn neutron stars could rotate initially at $P_0 \sim (20 - 30)$ ms (e.g., Xu et al. 2002).

It challenges the opinion above that the ages of a few supernova remnants are inconsistent with $T_{\rm c}$ of their related isolated stars (Table 1), which implies that some additional torque mechanisms do contribute to star braking. Among the five stars, three of them have $T_{\rm c} > 10T_{\rm SNR}$ (the age of supernova remnant), and other two $T_{\rm c} \lesssim 2T_{\rm SNR}$. In addition, electron cyclotron resonant lines are detected in two of the five neutron stars (1E 1207.4-5209:

Bignami et al. 2003; 1E 2259+568: Iwasawa et al. 1992), but the inferred magnetic fields, $B_{\rm cyc}$, from which are significantly smaller than that in the magnetic dipole radiation model, $B_{\rm d}$. The most prominent one in this age discrepancy issue is 1E 1207.4-5209, which has $T_{\rm c} \gtrsim 30 T_{\rm SNR}$ and $B_{\rm cyc} \lesssim B_{\rm d}/30$.

One probable and popularly-discussed way to solve the age problem is through an additional accretion torque (Marsden et al. 2001, Alpar et al. 2001, Menou et al. 2001). In this paper, whether an additional accretion torque can possibly solve the age discrepancy is investigated, including discussions about possible astrophysical implications.

2. The case of 1E 1207.4-5209

The key point in the age discrepancy of 1E 1207.4-5209 is how to spin down from $P_0 \sim 20$ ms to 424 ms in a short time of $T_{\rm SNR} \sim 7$ kyrs if its true age is $T_{\rm SNR}$. Certainly the problem disappears if one assumes a long initial period $P_0 \sim 400$ ms or a large braking index $n \sim 50$ (Pavlov et al. 2002); but this is not of Occam's razor since it is generally believed that rotation-powered radio pulsars born with ~ 20 ms, and brake with index $\lesssim 3$. May an additional accretion torque help the spindown? Actually, in an effort to reconcile $B_{\rm d}$ with $B_{\rm cyc}$, an accretion model for 1E 1207.4-5209 was proposed (Xu et al. 2003). However, a very difficulty in the model is how to choose a time-dependent accretion rate $\dot{M}_{\rm d}(t)$, and to determine the propeller torque with the rate $\dot{M}_{\rm d}$.

Nevertheless, the propeller phase works in the centrifugal inhibition regime when $r_{\rm m} > r_{\rm c}$; for a star with mass M and magnetic moment μ, the corotation radius $r_{\rm c} = [GM/(4\pi^2)]^{1/3} P^{2/3}$ and the magnetospheric radius $r_{\rm m} = [\mu^2/(\dot{M}_{\rm d}\sqrt{2GM})]^{2/7}$. To avoid the complex calculations of magnetohydrodynamics, the rotation energy loss due to propeller torque could be simply introduced as $\dot{E}_{\rm a} = -G\dot{M}_{\rm d}M/R_{\rm m}$, based on the energy conservation law. This is unphysical, but should be an limit for accretion braking. As $r_{\rm m}$ decreases ($\rightarrow r_{\rm c}^+$), $\dot{M}_{\rm d}$ increases, and $|\dot{E}_{\rm a}|$ increases too. Therefore the most efficient spindown (MESD) takes place when $r_{\rm m} \rightarrow r_{\rm c}^+$.

In a model where the propeller and electromagnetic torques are combined, in the MESD case, one can derive the period evolution

$$P < 1.1 B_{12}^2 R_6^4 (M/M_\odot)^{-2} (t/{\rm yrs}) + P_0 \ ({\rm ms}), \tag{1}$$

where $B_{12} = B/(10^{12}{\rm G})$ and $R_6 = R/(10^6{\rm cm})$. The right hand of Eq.(1) is an upper limit of P because 1, a realistic accretion rate may not be as high as that of MESD; and 2, the corresponding braking torque is not so effective. If 1E 1207.4-5209 is a conventional neutron star with mass $\sim M_\odot$ and radius $\sim 10^6$ cm, and the line features are related to cyclotron

absorptions near the surface (Xu et al. 2003; the polar magnetic field is thus 6×10^{10} G), one has $P < 3.8(t/\mathrm{kyrs}) + P_0$ (ms).

Therefore, assuming 1E 1207.4-5209 has a true age $t \sim 7$ kyrs and an initial period $P_0 \sim 20$ ms, the upper limit of the present period is ~ 40 ms ($\ll P = 434$ ms), and the age discrepancy can then not be solved in the conventional neutron star model. However, if 1E 1207.4-5209 is a strange star with low mass, for instance $R = 1$ km (and the mass is thus $\sim 10^{-3} M_\odot$, since low-mass strange stars have almost a homogenous density $\sim 4 \times 10^{14}\mathrm{g/cm}^3$; Alcock et al. 1986), the upper limit is then $P \simeq 110 B_{12}^2 (t/\mathrm{yrs}) + P_0$ (ms). In this case, 1E 1207.4-5209 could spin down to ~ 2.8 s during ~ 7 kyrs if its polar magnetic field 6×10^{10} G. In fact, the fitted radius of 1E 1207.4-5209 with a blackbody model is only $\sim$ 1km (Mereghetti et al. 1996; Vasisht et al. 1997) although a lager radius is possible if a light-element atmosphere is applied (Zavlin, Pavlov & Trümper 1998). The best-fit tow blackbody model of *XMM-Newton* data indicates an emitting radius ~ 3 km for the soft component with temperature ~ 200 eV (Bignami et al. 2003). Combined with its non-atomic feature spectrum, we may suggest that 1E 1207.4-5209 is a low-mass strange star with bare quark surface (Xu 2002, Xu et al. 2003).

An alternative possibility is that 1E 1207.4-5209 is a conventional neutron star, but the cyclotron resonant absorption forms far away from the surface. The polar magnetic field is $\sim 6 \times 10^{10}[(R + h)/R]^3$G if the resonant lines form at a height h. From Eq.(1), $424 < 1.1 \times 7 \times 10^3 B_{12}^2 + 20$, we estimate a low limit of the polar magnetic field to be $\sim 2.3 \times 10^{11}$ G. This implies that the resonant absorption region should be at a level of > 16 km height from the surface. Certainly, in case of no propeller torque (i.e., the polar magnetic field is $(1.7 - 3.6) \times 10^{12}$G), the height of resonant absorption region is $(30 - 40)$km.

It is worth noting that 1E 1207.4-5209 could be a low-mass neutron star with polar magnetic field $B_{12} = 0.06$. From Eq.(1) and the conditions of MESD, one has $R_6^2 > 4(M/M_\odot)$ for $P_0 \sim 20$ ms. This implies a neutron star with radius $R > 10$ km but mass $M < M_\odot$ (e.g., Shapiro & Teukolsky 1983). This result may have difficulties in explaining 1, a non-atomic spectrum (Xu et al. 2003), and 2, a possible small radius observed (Mereghetti et al. 1996; Vasisht et al. 1997; Bignami et al. 2003) and even the fitting result of a neutron star with 10 km and $1.4 M_\odot$ (Zavlin et al. 1998).

3. Other sources

If other sources list in Table 1 are neutron stars with $R_6 = 1$ and $M = M_\odot$, the low limits of polar magnetic fields are 6.1×10^{11} G for 1E 2259+586, 4.9×10^{10} G for PSR

B1757-24, 1.6×10^{11} G for PSR J1811-1925, and $(2.5 - 5.6) \times 10^{11}$ G for PSR J1846-0258. Among these sources, only possible cyclotron absorption is found in 1E 2259+586, and the limit field is within the range of that inferred from cyclotron line. This suggests that the cyclotron resonant may take place just above the stellar surface.

An interesting question is: how much mass could accrete during the propeller phase in case of MESD? Certainly, only a very small part of this matter can accrete onto the stellar surface. When MESD works, one obtains the accretion rate $\dot{M}_{\rm d} \sim 2^{11/6}\pi^{7/3}\mu^2(GM)^{-5/3}P^{-7/3}$ from $r_{\rm m} \simeq r_{\rm c}$. If the quantities are re-scaled, $m = M_{\rm d}/M_\odot$, $\tau = t/{\rm yr}$, and $p = P/{\rm ms}$, one has ${\rm d}m/{\rm d}\tau \sim 0.24\ \mu_{30}^2(M/M_\odot)^{-5/3}p^{-7/3}$. Combining with Eq.(1), one comes to

$$m < \int_0^\infty {\rm d}\tau = 0.16(M/M_\odot)^{-2/3}I_{45}p_0^{-4/3}, \tag{2}$$

where $p_0 = P_0/{\rm ms}$. Note that the upper limit of accretion mass, $M_{\rm d}$, in the right hand of Eq.(2) does not depend on the magnetic momentum. Typically for $p_0 = 20$, the upper limit of accretion mass is $2.95 \times 10^{-3}M_\odot$, which is reasonable since the amount of the fall-back material after supernova explosion could be as high as $0.1M_\odot$ (Lin, Woosley & Bodenheimer 1991; Chevalier 1989). Due to r-mode instability, a nascent neutron star may loss rapidly its angular momentum though gravitational radiation if the initial period is less than $\sim 3 - 5$ ms (Andersson & Kokkotas 2001). The upper limit of accretion mass in case of MESD is $\sim 0.04M_\odot$ for $p_0 = 3$. These results indicate that the fall-back matter is enough to brake the center stars by propeller torque in MESD case.

4. A model with self-similar accretion rate

Though the study about MESD torque provides some useful information on the accretion model, including the appropriate magnetic field and the mass of fall-back disk around a neutron star, the accretion rate of MESD torque is questionable in realistic cases. After a dynamical time a fossil disk may form. For a viscosity-driven disk, the accretion could be in a self-similar way, with an accretion rate of (Cannizzo, Lee & Goodman 1990)

$$\dot{m} = \dot{m}_0,\ 0 < t < T;\quad \dot{m} = \dot{m}_0(t/T)^{-\alpha},\ t \geq T, \tag{3}$$

where T is of order the dynamical time, $\dot{m} = {\rm d}m/{\rm d}t$. Assuming $T \sim 1$ms, an initial disk mass $\sim 0.006M_\odot$, and an opacity dominated by electron scattering ($\alpha = 7/6$), Chatterjee, Hernquist and Narayan (2000) develop a first detailed model of fossil disk accretion for anomalous X-ray pulsars (AXPs). However it is noted by Francischelli and Wijers (2002) that Kramers opacity may prevail in the fossil disk (i.e., $\alpha = 1.25$). In the regime of conventional

neutron stars, we will calculate the accretion torque through the realistic accretion rate of Eq.(3), assuming $\alpha = 1.25$ and $T = 1$ms, with the inclusion of magnetic dipole radiation. The spinup/spindown torque proposed by Menou et al. (1999),

$$\dot{J} = 2\dot{M}_{\rm d} r_{\rm m}^2 \Omega_{\rm k}(r_{\rm m})[1 - \Omega/\Omega_{\rm k}(r_{\rm m})], \tag{4}$$

is applied for the action of fossil disk in the model, where $\Omega = 2\pi/P$, $\Omega_{\rm k}(r_{\rm m})$ is the Keplerian angular velocity at the magnetospheric boundary.

One may compute the accretion rate, $\dot{m}$, as well as the spin evolution $P(t)$. The total disk mass could be $m = \int_0^\infty \dot{m} dt$. It is worth noting that the disk mass obtained in this way could be much larger than that in MESD case because of the inclusion of the high accretion in the initial period.

We think that the accretion rate characterized by Eq.(3) is on average in a sense. The period derivative, $\dot{P}$, may be affected by dynamical instabilities or some stochastic processes, whereas the period, P, is of the integration over a very long time. We therefore calculate $P(T_{\rm SNR})$ for any disk mass, m, and polar magnetic field, B_{12}, of neutron stars. For 1E 1207.4-5209, the calculated contour of period relative error, $|P(T_{\rm SNR}) - P|/P$ ($P = 0.424$s for 1E 1207.4-5209), is shown in Fig.1. A reasonable parameter set (disk mass m and polar magnetic field B_{12}) is chosen if the following criteria are met: 1, the period relative error is smaller; 2, $m < 0.1$; 3, $B_{12,\rm cyc} < B_{12} < B_{12,\rm d}$ (Table 1). We then have $m = 0.054$ and $B_{12} = 3.55$. The parameter sets for other sources can also be obtained in this way, which are listed on Fig.2 (except for PSR J1811-1925). The period evolution curves, with these parameters, are drawn in Fig.2. Note these curves do not change significantly if the parameter sets shift reasonably.

The heights, h, of cyclotron resonant scattering regions can be obtained, based on the differences of parametric magnetic field, B_{12}, and the field inferred from absorption features, $B_{12,\rm cyc}$. It is found that $h \sim 29$ km and $\sim (8.8 - 15)$ km for 1E 1207.4-5209 and 1E 2259+586, respectively, in the model.

Whether or not the disk will influence the spindown of the neutron star or suppress the radio emission will depend on the location of $r_{\rm m}$ relative to the light cylinder radius, $r_{\rm L}$, and the corotation radius $r_{\rm c}$ (Chatterjee et al. 2000). Magnetic dipole radiation dominates, and the disk and the star will effectively evolve independently if $r_{\rm m} > r_{\rm L}$; but in other cases, accretion onto the star will lead to accretion-induced X-ray emission with radio quiet. We see from Fig.2 that the condition of $r_{\rm m} > r_{\rm L}$ is satisfied only for PSR B1757-24 when it is older than $\sim 10^3$ years. We are therefore not surprise that PSR B1757-24 is now radio loud whereas the others (1E 1207.4-5209, 1E 2259+586, and PSR J1846-0258) are radio quiet. The AXP 1E 2259+586 is in a tracking phase, and we expect other two (1E 1207.4-5209 and

PSR J1846-0258) will evolve to be AXPs when they are in tracking phases too.

PSR J1811-1925 is an interesting exception among the five sources, whose age is certain if it has physical association with the remnant of a supernova recorded in A.D. 386. In its calculated contour, we can only choose $\{B_{12} = 2.6, m = 0.1\}$ or $\{B_{12} = 1.6, m = 2.2\}$; both the parameter sets are not reasonable (i.e., can not meet those 3 criteria). This may imply that the accretion of PSR J1811-1925 is not self-similar. Recalling that the low limit of polar field is only 1.6×10^{11}G if in MESD case, we could suggest that PSR J1811-1925 has a field within $(1.6 - 17) \times 10^{11}$G, with an accretion stronger than that of Eq.(3) but weaker than that in MESD case. This result hints that PSR J1811-1925 is radio quiet (Crawford et al. 1998). In addition, the parametric field, $B_{12} = 3.55$, chosen for 1E 1207.4-5209, which is close to $B_{12,\rm d} = 3.6$, would also indicate that the real accretion is not described by Eq.(3). In fact the accretion of Eq.(3) is for the capture of material by black holes where magnetic field is not important, which could differ from that for neutron stars with strong fields.

5. Conclusions and Discussions

The possibility of solving the age discrepancy by an accretion-assisted torque is discussed. We find that: 1, 1E 1207.4-5209 can not be a neutron star, but a low-mass bare strange star, if the cyclotron resonant region is near the polar cap with a magnetic field of 6×10^{10}G; whereas it could be a conventional neutron star if the cyclotron lines form at a height of $(16-40)$km. An identification of smaller radius or low altitude cyclotron formation favors a low-mass bare strange star model for 1E 1207.4-5209. 2, Among the five sources with age problems, the magnetic dipole radiation dominates the evolution of PSR B1757-24 at present, and the others are in propeller (or tracking) phases. 3, The real accretion around these sources may differ from a self-similar one (Eq.(3)), at least for PSR J1811-1925. 4, By a calculation with self-similar accretion, it is suggested that PSR J1846-0258 and 1E 1207.4-5209 (and probably PSR J1811-1925) would evolve to be anomalous X-ray pulsars in the future.

The debris disks formed following supernovae are currently conjectured also for interpreting other astrophysical phenomena, e.g, anomalous X-ray pulsars and soft $\gamma-$ray repeaters. Factually, these disks around the sources could be bright in a wide spectral range. Recent discoveries of possible optical and near-infrared emission from a few AXPs may be a hint of such kind fallback accretion disks (1E 2259+586: Hulleman et al. 2001; 1RXS J170849-400910: Israel et al. 2002; 1E 1048.1-5937: Wang & Chakrabarty 2002). Although a comparison of optical and near-infrared observations with theoretical predictions of spectra of disks around neutron stars (Perna et al. 2000) have helped rule out the presence of

disks in some cases, more detailed studies in this aspect is still necessary, which may be an effective way to test the fossil-disk model for young neutron stars.

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Table 1. List of neutron stars with age discrepancies

Stars	SNRs	P(s)	$\dot{P}(10^{-13}\mathrm{s/s})$	$T_{\mathrm{c}}(10^3\mathrm{y})$	$T_{\mathrm{SNR}}(10^3\mathrm{y})$	$B^*_{12,\mathrm{d}}$	$B^{\S}_{12,\mathrm{cyc}}$	Ref.
1E 1207.4-5209	PKS 1209-51/52	0.424	0.07-0.3	200-900	~ 7	1.7-3.6	0.06	1,2
1E 2259+586†	CTB 109	6.98	4.84	228	17	59	0.4-0.9	3,4,5
PSR B1757-24	G5.4-1.2	0.125	1.28	16	> 39	4.0	—	6
PSR J1811-1925	G11.2-0.3	0.065	0.44	24	1.6	1.7	—	7
PSR J1846-0258	Kes 75	0.325	71	0.72	0.9-4.3	49	—	8

*The magnetic fields in the magnetic dipole radiation model: $B_{\mathrm{d}} = 3.2 \times 10^{19}\sqrt{P\dot{P}}$ G, $B_{12,\mathrm{d}} = B/(10^{12}\mathrm{G})$.

§The magnetic fields (in unit of 10^{12} G) derived from spectral features as cyclotron resonant scattering. No gravitation redshift is included here.

†An anomalous X-ray pulsar.

References. — 1, Pavlov et al. (2002); 2, Bignami et al. (2002); 3, Gavriil & Kaspi (2002); 4, Hughes et al. (1981); 5, Iwasawa et al. (1992); 6, Marsden et al. (2001); 7, Torii et al. (1999); 8, Gotthelf et al. (2000).

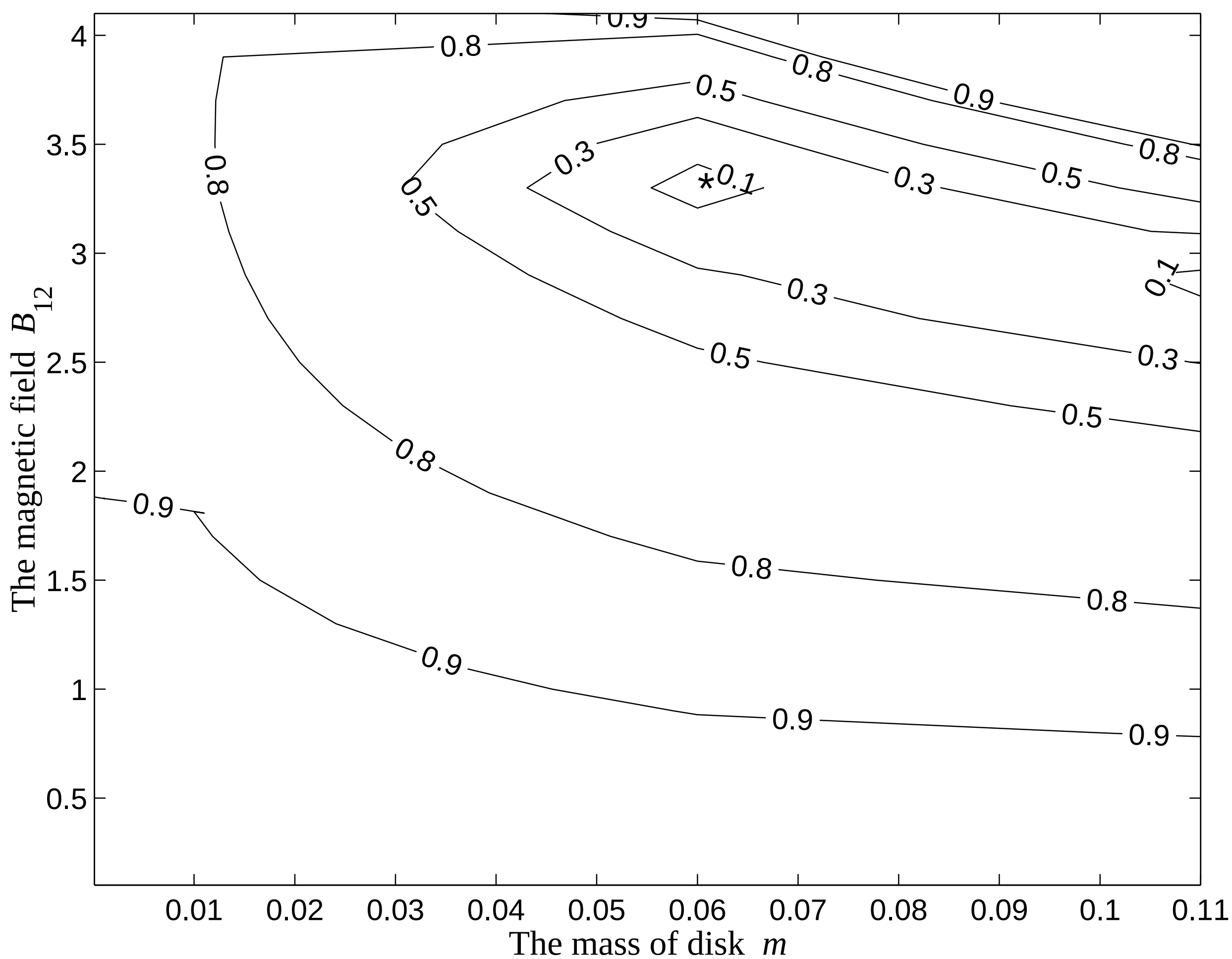

Fig. 1.— The contour of the relative error of period for 1E1207.4-5209 in a model with self-similar accretion. The star sign indicates the parametric position we choose for a reasonable neutron star and the fossil disk around it. The disk mass m is in $M_\odot$ and the polar magnetic field B_{12} in 10^{12}G.

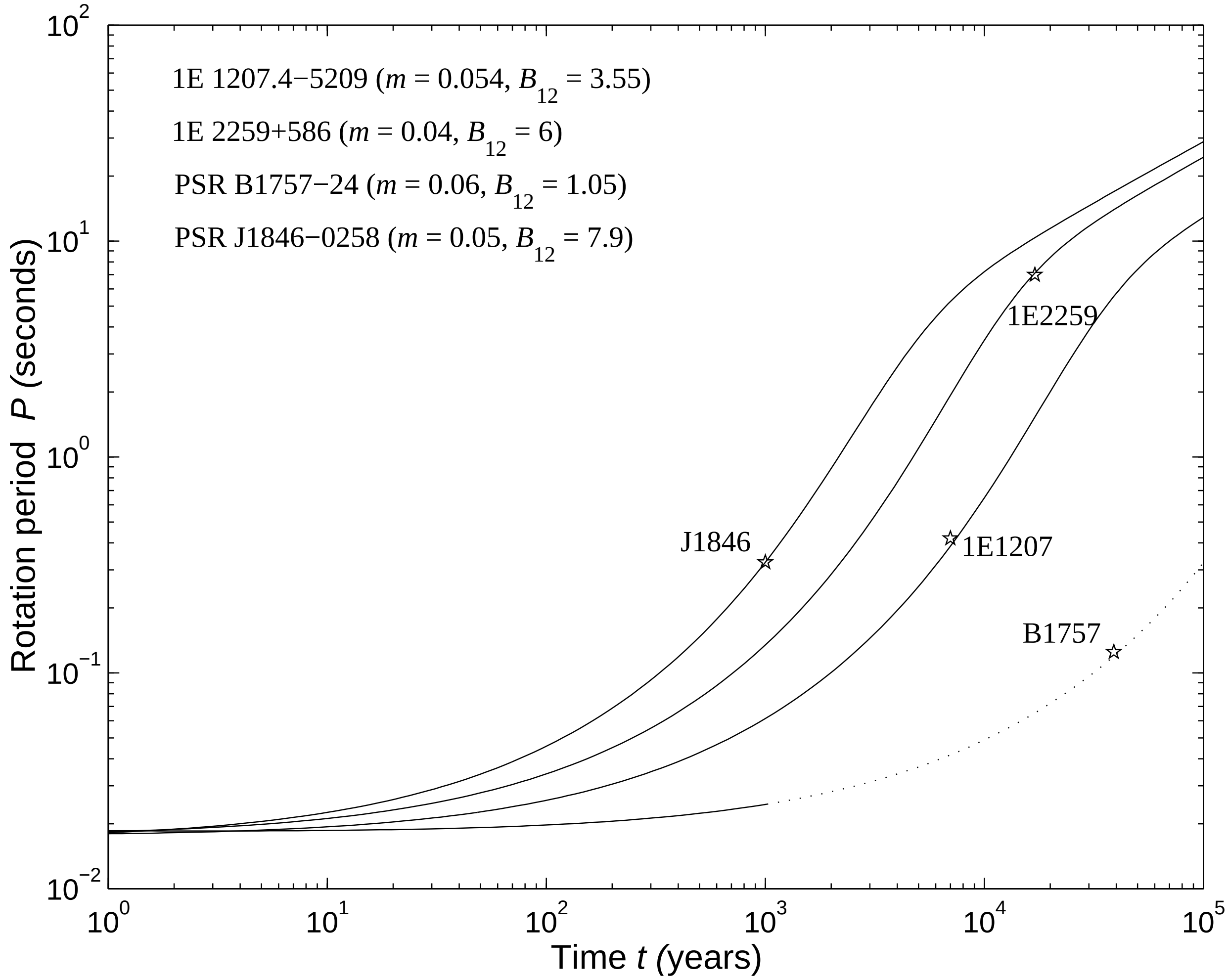

Fig. 2.— The period evolution in a model with self-similar accretion for the four pulsars labelled. The parameter sets used to calculate the curves are also listed. The solid part of the curves indicates that the accretion torque works (i.e., the magnetospheric radius is smaller than that of light cylinder, $r_{\rm m} < r_{\rm L}$), while the dashed part of PSR B1757-24 means that the pulsar and the fossil disk could evolve independently (i.e., $r_{\rm m} > r_{\rm L}$).